International Journal of Engineering, Science and Mathematics (UGC Approved)
Vol. 6 Issue 8, December 2017, ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Bridges and cut-vertices of Intuitionistic Fuzzy Graph Structure

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Abstract

In this paper,, the concept of bridge and cut vertices in an intuitionistic fuzzy graph structures (IFGS) are defined and their properties are studied. We describe the existence of bridge in an IFGS and obtain some equivalent conditions. Also intuitionistic fuzzy bridges and intuitionistic fuzzy cut vertices are characterized using partial intuitionistic fuzzy spanning subgraph structures..

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Key words:

Intuitionistic fuzzy graph structure;
B_i –Bridges;
B_i -Cut-vertices.

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2010 Mathematics Subject Classification: 03F55, 05C05, 05C72, 05C38.

1. Introduction:

The idea of fuzzy sets was originated by L.A. Zadeh [14] in 1965. A. Rosenfeld [9] commenced

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the idea of fuzzy relation and fuzzy graph and developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. The notion of graph G = (V, E) to graph structure G = (V, R_1, R_2,......., R_k) was generalized by E. Sampatkumar in [11]. The overview of fuzzy graph structure was later discussed by T. Dinesh and T. V. Ramakrishnan [2]. M. G. Karunambigai, O. K. Kalaivani in [3] defined the bridge of IFG. Sheik Dhavudh, R. Srinivasan in [10] discussed the cutvertices of IFG.

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2. Preliminaries:

In this section, we review some definitions that are necessary in this paper which are mainly taken from [2], [3], [11], [12] and [13].

Definition (2.1): Let G = ($V,R_1,R_2,...,R_k$) be a graph and let A be an intuitionistic fuzzy subset on V and B_1, B_2 ,...,B_k are intuitionistic fuzzy relations on V which are mutually disjoint symmetric and irreflexive respectively such that

$$\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v) \text{ and } \nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v) \quad \forall \text{ u, v } \in V \quad \text{and i = 1,2,..., k.}$$

Then $\tilde{G}$ = (A, B_1,B_2,…..,B_k) is an intuitionistic fuzzy graph structure of G.

Definition (2.2): Let $\tilde{G}$ = (A, B_1,B_2,...,B_k) be an intuitionistic fuzzy graph structure of a graph structure G=(V,R_1,R_2,...,R_k), then $\tilde{H}$ =(A,C_1,C_2,...,C_k) is called a partial intuitionistic fuzzy spanning subgraph structure of $\tilde{G}$ =(A,B_1,B_2,.....,B_k) if $\mu_{C_r}(u,v) \le \mu_{B_r}(u,v)$ and $\nu_{C_r}(u,v) \le \nu_{B_r}(u,v)$ for r =1,2,...,k and ∀ u,v ∈V, uv ∈ B_i and i=1,2,..., k.

Note(2.3): Throughout this paper, unless otherwise specified $\tilde{G}$ = (A,B_1,B_2,...,B_k) will represent an intuitionistic fuzzy graph structure with respect to graph structure G = (V, R_1,R_2,...,R_k) and B_i, for i =1, 2,..., k will refer to the number of intuitionistic fuzzy relations on V.

Definition (2.4): Let $\tilde{G}$ be an IFGS of a graph structure G. If (u,v) ∈ supp(B_i)= { (u,v) ∈ $V \times V$: μ_{B_i} (u,v) > 0 , ν_{B_i} (u,v) < 1}, then (u,v) is said to be a B_i−edge of $\tilde{G}$.

Definition (2.5): In an IFGS $\tilde{G}$, B_i -path is a sequence of vertices $u_0,u_1,...,u_n$ which are distinct (except possibly $u_0 = u_n$) such that (u_{j-1},u_j) is a B_i-edge for all j = 1,2,...,n.

Definition (2.6): In an IFGS $\tilde{G}$,a path is a sequence of vertices v_1,v_2 ,......, v_n ($\in V$) which are distinct (except possibly $v_1= v_n$) such that (v_j, v_{j+1}) is a B_j -edge for some j = 1,2,…, n and i = 1,2,3, ..., k.

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Definition (2.7): In an IFGS $\tilde{G}$, the μ_{B_i}-strength of a B_i-path $u_0,u_1,...,u_n$ is denoted by $S_{\mu_{B_i}}$ and is the min $\mu_{B_i}(u_{j-1},u_j)$ for j=1,2,…,n. i.e. $S_{\mu_{B_i}} = \bigwedge_{j=1}^{n} \mu_{B_i}(u_{j-1},u_j)$ for i=1,2,...,k.

Definition (2.8): In an IFGS $\tilde{G}$, **t**he ν_{B_i}-strength of a B_i-path $u_0,u_1,...,u_n$ is denoted by $S_{\nu_{B_i}}$ and is the max $\nu_{B_i}(u_{j-1},u_j)$ for j=1,2,…,n. i.e. $S_{\nu_{B_i}} = \bigvee_{j=1}^{n} \nu_{B_i}(u_{j-1},u_j)$ for i=1,2,...,k.

Definition (2.9): **T**he strength of a B_i-path $u_0,u_1,...,u_n$ in an IFGS $\tilde{G}$ is denoted by S_{B_i} and is defined as $S_{B_i} = \left(\bigwedge_{j=1}^{n} \mu_{B_i}(u_{j-1},u_j), \bigvee_{j=1}^{n} \nu_{B_i}(u_{j-1},u_j)\right)$ for i=1,2,...,k.

Definition (2.10): The strength S of a path in an IFGS $\tilde{G}$ is the weight of the weakest edge of the path. i.e., strength of path = $S = \left(\min_{i=1}^{k}\left(S_{\mu_{B_i}}\right), \max_{i=1}^{k}\left(S_{\nu_{B_i}}\right)\right)$.

Definition (2.11): In any IFGS $\tilde{G}$,

$\mu_{B_i}^{2}(u,v) = \mu_{B_i} \circ \mu_{B_i}(u,v) = \text{Max}\{\mu_{B_i}(u,w) \wedge \mu_{B_i}(w,v)\}$ and

$\mu_{B_i}^{j}(u,v) = (\mu_{B_i}^{j-1} \circ \mu_{B_i})(u,v)$, j=2,3,...,m for any $m \geq 2$.

Also $\mu_{B_i}^{\infty}(u,v) = \bigvee_{j=1}^{\infty} \mu_{B_i}^{j}(u,v)$.

Definition (2.12): In any IFGS $\tilde{G}$,

$\nu_{B_i}^{2}(u,v) = \nu_{B_i} \circ \nu_{B_i}(u,v) = \text{Min}\{\nu_{B_i}(u,w) \vee \nu_{B_i}(w,v)\}$ and

$\nu_{B_i}^{j}(u,v) = (\nu_{B_i}^{j-1} \circ \nu_{B_i})(u,v)$, j=2,3,...,m for any $m \geq 2$.

Also $\nu_{B_i}^{\infty}(u,v) = \bigwedge_{j=1}^{\infty} \nu_{B_i}^{j}(u,v)$.

Definition (2.13): In an IFGS $\tilde{G}$, a B_i-cycle is an alternating sequence of vertices and edges $u_0,e_1,u_1,e_2,...,u_{n-1},e_n,u_n = u_0$ consisting only of B_i-edges.

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Definition (2.14): An IFGS $\tilde{G}$ is a B_i-forest if the subgraph structure induced by B_i -edges is a forest, i.e., if it has no B_i -cycles.

Result (2.15): $\tilde{G}$ is a B_i-tree when it is a B_i–connected B_i–forest.

Definition (2.16): $\tilde{G}$ is an intuitionistic fuzzy B_i-forest if it has a partial intuitionistic fuzzy spanning sub-graph structure $\tilde{H}_i = (A,C_1,C_2,...,C_k)$ which is a C_i-forest where for all B_i-edges not in H_i , $\mu_{B_i}(x,y) < \mu_{C_i}{}^{\infty}(x,y)$ and $\nu_{B_i}(u,v) < \nu_{C_i}{}^{\infty}(x,y)$.

Definition (2.17): $\tilde{G}$ is an intuitionistic fuzzy B_i-tree if it has a partial intuitionistic fuzzy spanning sub-graph structure $\tilde{H}_i = (A,C_1,C_2,...C_k)$ which is a C_i-tree where for all B_i-edges not in $\tilde{H}_i$, $\mu_{B_i}(x,y) < \mu_{C_i}{}^{\infty}(x,y)$ and $\nu_{B_i}(u,v) < \nu_{C_i}{}^{\infty}(x,y)$.

Theorem (2.18): Let $\tilde{G}$ be a B_i-cycle. $\tilde{G}$ is an intuitionistic fuzzy B_i-cycle iff $\tilde{G}$ is not an intuitionistic fuzzy B_i-tree.

3. B_i-Bridges and B_i-Cut-vertices of IFGS

Definition (3.1): An edge (u,v) is said to be a B_i-bridge in an IFGS $\tilde{G}$ if either $\mu'_{B_i}{}^{\infty}(u,v) < \mu_{B_i}{}^{\infty}(u,v)$ and $\nu'_{B_i}{}^{\infty}(u,v) \geq \nu_{B_i}{}^{\infty}(u,v)$ or $\mu'_{B_i}{}^{\infty}(u,v) \leq \mu_{B_i}{}^{\infty}(u,v)$ and $\nu'_{B_i}{}^{\infty}(u,v) > \nu_{B_i}{}^{\infty}(u,v)$.

In other words, deleting an edge (u,v) reduces the B_i -strength of connectedness between some pair of vertices or (u,v) is a B_i –bridge if there exists vertices x and y s.t. (u,v) is an edge of every strongest path from x to y.

Definition (3.2): If an IFGS $\tilde{G}$ has at least one B_i-bridge, $\tilde{G}$ is said to have a bridge.

Theorem (3.3): (i) If there exists one B_i, (i =1,2,..,k) which is constant then $\tilde{G}$ has no B_i-bridge.

(ii) If there exists one B_i, (i =1,2,..,k) which is not constant then $\tilde{G}$ has a B_i-bridge.

Proof: (i) Suppose that all B_i (i = 1, 2, …., k) are constant.

Let $\mu_{B_i}(u, v) = c$ and $\nu_{B_i}(u, v) = d$ $\forall$ u, v $\in V_i$ where $0 \leq c, d \leq 1$.

Since the degree of membership of each B_i–edge are same (i.e., c) and degree of non- membership of each B_i–edge are also same (i.e., d).

Therefore, deleting any edge does not reduce the strength of connectedness between any pair of vertices.

Hence $\tilde{G}$ has no B_i-bridge.

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(ii) Assume that B_i is not constant. Choose an edge $(u_x, v_x) \in V \times V$ such that $\mu_{B_i}(u_x, v_x) = \max\{\mu_{B_i}(u, v) : \forall (u, v) \in V \times V\}$ and $\nu_{B_i}(u_x, v_x) = \min\{\nu_{B_i}(u, v) : \forall (u, v) \in V \times V\}$.

Since $\mu_{B_i}(u_x,v_x) > 0$ and $\nu_{B_i}(u_x,v_x) < 1$ therefore, there exists atleast one B_i- edge (u_y, v_y) distinct from (u_x,v_x) such that $\mu_{B_i}(u_y,v_y) < \mu_{B_i}(u_x,v_x)$ and $\nu_{B_i}(u_y,v_y) > \nu_{B_i}(u_x,v_x)$.

$\therefore$ If we delete the B_i-edge (u_x,v_x) , then the strength of connectedness between u_x and v_x in the fuzzy subgraph structure thus obtained is decreased.

i.e., $\mu'^{\infty}_{B_i}(u_x,v_x) < \mu_{B_i}(u_x,v_x)$ and $\nu'^{\infty}_{B_i}(u_x,v_x) > \nu_{B_i}(u_x,v_x)$.

$\therefore$ (u_x,v_x) is a B_i–bridge of $\tilde{G}$. (by definition of B_i–bridge.)

Theorem (3.4): In an IFGS $\tilde{G} = (A, B_1,B_2,...,B_k)$, after deleting a B_i-edge (u,v) , we have an IFGS $\tilde{G}' = (A,B'_1,B'_2,\ldots,B'_k)$ of vertices (u_x,v_x) for (x,y =1,2,...,n) then the following conditions are equivalent:

(i) $\mu'^{\infty}_{B_i}(u,v) < \mu_{B_i}(u,v)$ and $\nu'^{\infty}_{B_i}(u,v) > \nu_{B_i}(u,v)$.

(ii) (u,v) is a B_i-bridge.

(iii) (u,v) is a not a B_i-edge of any cycle.

Proof: To Prove (i) $\Rightarrow$(ii).

Given that $\mu'^{\infty}_{B_i}(u,v) < \mu_{B_i}(u,v)$ and $\nu'^{\infty}_{B_i}(u,v) > \nu_{B_i}(u,v)$.

To prove (u,v) is a B_i-bridge.

Suppose that (u,v) is not a B_i-bridge, then

$\mu'^{\infty}_{B_i}(u_x,v_y) = \mu^{\infty}_{B_i}(u,v) \geq \mu_{B_i}(u,v)$ and $\nu'^{\infty}_{B_i}(u_x,v_y) = \nu^{\infty}_{B_i}(u,v) \leq \nu_{B_i}(u,v)$

which contradicts (i).

$\Rightarrow$ (u,v) is a B_i-bridge.

To Prove (ii) $\Rightarrow$(iii).

Given (u,v) is a B_i-bridge.

To Prove (u,v) is a not a B_i-edge of any cycle.

Suppose (u,v) is a a B_i-edge of any cycle,

$\Rightarrow$ any path which has a B_i-edge (u,v) with the use of rest of the cycle as a path from u to v

which is a contradiction to our assumption.

$\therefore$(u,v) is a not a B_i-edge of any cycle.

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To Prove (iii) $\Rightarrow$(i).

Let (u,v) is a not a B_i-edge of any cycle.

To Prove $\mu'^{\infty}_{B_i}$(u,v)< μ_{B_i}(u,v) and $\nu'^{\infty}_{B_i}$(u,v) > ν_{B_i}(u,v).

Suppose $\mu'^{\infty}_{B_i}$(u_x,v_y) $\geq$ μ_{B_i}(u,v) and $\nu'^{\infty}_{B_i}$(u_x,v_y) $\leq$ ν_{B_i}(u,v).

Then there exists a path from u to v which does not involve (u,v) that has strength greater than or equal to μ_{B_i}(u,v) and less than or equal to ν_{B_i}(u,v).

Also this path together with (u,v) form a cycle , which is a contradiction to our assumption.

$\Rightarrow$ $\mu'^{\infty}_{B_i}$(u,v)< μ_{B_i}(u,v) and $\nu'^{\infty}_{B_i}$(u,v) > ν_{B_i}(u,v).

Hence (i), (ii) and (iii) are equivalent.

Theorem (3.5): If (u,v) is a B_i-bridge of an IFGS $\tilde{G}$ = (A, B_1,B_2,...,B_k) and $\tilde{H}$ = (A,B'_1,B'_2,…,B'_k) is a partial intuitionistic fuzzy spanning subgraph structure obtained by deleting (u,v) for i=1,2,...,k. Then $\mu'^{\infty}_{B_i}$(u,v) < μ_{B_i}(u,v) and $\nu'^{\infty}_{B_i}$(u,v) > ν_{B_i}(u,v).

Proof: If possible, Suppose there exists a B_i-path of strength greater than μ_{B_i}(u,v) and less than ν_{B_i}(u,v) from u to v not having the B_i-edge (u,v).

i.e., suppose $\mu'^{\infty}_{B_i}(u,v) \geq \mu_{B_i}(u,v)$ and $\nu'^{\infty}_{B_i}(u,v) \leq \nu_{B_i}(u,v)$.

$\Rightarrow$ Any B_i-path which contains B_i-edge (u,v) can be replaced by a B_i-path which does not have B_i-edge (u,v) and its strength is not reduced. This contradicts that (u,v) is a B_i-bridge of $\tilde{G}$. Thus $\mu'^{\infty}_{B_i}$(u,v) < μ_{B_i}(u,v) and $\nu'^{\infty}_{B_i}$(u,v) > ν_{B_i}(u,v) for i =1,2,...,k.

Corollary (3.6): Converse of the above theorem is also true. i.e., if $\mu'^{\infty}_{B_i}$(u,v) < μ_{B_i}(u,v) and $\nu'^{\infty}_{B_i}$(u,v) > ν_{B_i}(u,v), then (u,v) is a B_i-bridge of $\tilde{G}$.

Theorem (3.7): Let $\tilde{G}$ be an intuitionistic fuzzy graph structure which is an intuitionistic fuzzy B_i-forest. Then the B_i–edge of the partial intuitionistic fuzzy spanning subgraph structure $\tilde{H}_i$ = (A, C_1, C_2,...,C_k) which is a C_i- forest, are the B_i-bridges of $\tilde{G}$.

Proof: Two cases arises.

Case I: (u,v) is a B_i-edge which does not belong to $\tilde{H}_i$.

By definition of an intuitionistic fuzzy B_i-forest,

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μ_{B_i}(u,v)< $\mu_{C_i}{}^{\infty}$(u,v)≤ $\mu_{B_i}{}^{\infty}$(u,v) and ν_{B_i}(u,v) > $\nu_{C_i}{}^{\infty}$(u,v) ≥ $\nu_{B_i}{}^{\infty}$(u,v) where (A,B′₁,B′₂,…,B′ₖ) be a partial intuitionistic fuzzy spanning subgraph structure obtained by deleting (u,v).

∴ by theorem (3.5) , (u,v) is not a B_i-bridge.

Case II: (u,v) is a C_i-edge which belongs to $\tilde{H}_i$

If possible, suppose (u,v) is not a B_i-bridge,

∴ there exists a B_i-path P_i from u to v not having (u,v) with strength greater than or equal to μ_{B_i}(u,v) and less than or equal to ν_{B_i}(u,v).

$$\therefore\ \mu'^{\infty}_{B_i}(u,v)=\mu_{B_i}{}^{\infty}(u,v)\geq\mu_{B_i}(u,v)\ \text{and}\ \nu'^{\infty}_{B_i}(u,v)=\nu_{B_i}{}^{\infty}(u,v)\leq\nu_{B_i}(u,v).$$

∴ P_i and $\tilde{H}_i$ form B_i-cycle. But $\tilde{H}_i$ does not contain C_i-cycle,

∴ P_i contains B_i-edge not in $\tilde{H}_i$.

Let (x,y) be a B_i-edge of P_i.

∴ By definition of an intuitionistic fuzzy B_i-forest, it can be replaced by a C_i-path in $\tilde{H}_i$ which has strength greater than μ_{B_i}(x,y) and less than ν_{B_i}(x,y).

Also $\mu_{B_i}(x,y)\geq\mu_{B_i}(u,v)$ and $\nu_{B_i}(x,y)\leq\nu_{B_i}(u,v)$.

All C_i-edges of P_i are stronger than μ_{B_i}(x,y) and ν_{B_i}(x,y) which is greater than or equal to μ_{B_i}(u,v) and less than or equal to ν_{B_i}(u,v).

Thus P_i does not have (u,v).

If it contains (u,v), its strength will be less than or equal to μ_{B_i}(u,v) and greater than or equal to ν_{B_i}(u,v), i.e., $\mu_{C_i}(u,v)\leq\mu_{B_i}(u,v)$ and $\nu_{C_i}(u,v)\geq\nu_{B_i}(u,v)$.

⇒ there exists a C_i-path in $\tilde{H}_i$ from u to v not having (u,v).

⇒ there exists a C_i-cycle in $\tilde{H}_i$.

And thus there exists a B_i-cycle which is not possible.

∴(u,v) is a B_i-bridge.

Hence B_i–edge of $\tilde{H}_i$ are the B_i-bridges of $\tilde{G}$.

Definition (3.8): $\tilde{G}$=(A_1,B′₁,B′₂,...,B′ₖ) is the partial intuitionistic fuzzy subgraph structure obtained by removing a vertex w of $\tilde{G}$, i.e.,

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μ'_{A_i} (w) = 0 and μ'_{A_i} (u) = μ_{A_i} (u) $\forall$u ≠ w, μ'_{B_i} (w,v) =0 and ν'_{B_i} (w,v) = 0 $\forall$ v ∈ V and

μ'_{B_i} (u,v) = μ_{B_i} (u,v) and ν'_{B_i} (u,v) = ν_{B_i} (u,v) $\forall$(u,v) ≠ (w,v), i=1,2,...,k.

Definition (3.9): A vertex w of $\tilde{G}$ is a μ_{B_i}-cut vertex if deleting it reduces the μ_{B_i}- strength of connectedness between some pair of vertices.

Definition (3.10): A vertex w of $\tilde{G}$ is a ν_{B_i}-cut vertex if deleting it reduces the ν_{B_i}- strength of connectedness between some pair of vertices.

Definition (3.11): A vertex w is said to be a B_i –cut vertex of intuitionistic fuzzy graph structure $\tilde{G}$ if deleting a vertex w reduces the B_i - strength of connectedness between some pair of vertices. In other words, if either $\mu'^{\infty}_{B_i}$(u,v) < $\mu^{\infty}_{B_i}$ (u,v) and $\nu'^{\infty}_{B_i}$(u,v) ≥ $\nu^{\infty}_{B_i}$(u,v) or $\mu'^{\infty}_{B_i}$(u,v) ≤ $\mu^{\infty}_{B_i}$(u,v) and $\nu'^{\infty}_{B_i}$(u,v) > $\nu^{\infty}_{B_i}$(u,v) for some u,v∈V.

Now we discuss some results on B_i -bridges and B_i -cut vertices.

Theorem (3.12): Let $\tilde{G}$ be an IFGS with $\tilde{G}$*= (supp(A), supp(B_1), supp(B_2),..., supp(B_k)) a B_i -cycle. If a vertex of $\tilde{G}$ is a B_i –cut vertex of $\tilde{G}$, then it is a a common vertex of two B_i-bridges.

Proof: Consider a B_i -cut vertex w of $\tilde{G}$. By the definition of a B_i-cut vertex, there exists two vertices u and v different from w such that w is on every strongest u−v B_i -path.

Given that $\tilde{G}$ is a B_i-cycle.

then there exists only one strongest B_i - path P_i from u to v containing w.

All B_i -edges of P_i are B_i -bridges. So w is common to two B_i -bridges.

Converse of the above result is also true as is apparent from the next theorem:

Theorem (3.13): Let $\tilde{G}$ be an IFGS. If w is common to at least two B_i -bridges of $\tilde{G}$, then w is a B_i-cut vertex.

Proof: Let (u_1,w) and (w, v_2) be two B_i -bridges with w as the common vertex.

Since (u_1,w) is a B_i -bridge, it is on every strongest u-v B_i-path for some u and v.

Case I: w ≠ u, w ≠ v^i

In this case, w is on every strongest u -v B_i -path for some u and v. Then w is a B_i -cut vertex.

Case II: Either w = u or w = v

In this case either (u_1,w) is on every strongest u -w B_i -path or (w, v_2) is on every strongest w- v B_i -path.

If possible, let w be not a B_i -cut vertex.

By definition of B_i -cut vertex, there exists a strongest B_i -path not containing w between any pair of vertices.

Consider such a path P_i joining u_1 and v_2. Then P_i ,(u_1,w), (w, v_2) form a B_i -cycle.

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Subcase (i): Let u_1,w, v_2 be not a strongest B_i -path.

Then (u_1,w) or (w,v_2) or both become the weakest B_i-edges of the above B_i–cycle consisting of P_i ,(u_1,w) and (w, v_2) since every B_i -edge of P will be stronger than (u_1,w) and (w, v_2) .

This is not possible since (u_1,w) and (w, v_2) are B_i-bridges.

Subcase (ii): Let u_1, w,v_2 also be a strongest B_i-path joining u_1,v_2

$\mu_{B_i}{}^{\infty}(u_1, v_2) = \mu_{B_i}(u_1,w) \wedge \mu_{B_i}(w,v_2)$ and $\nu_{B_i}{}^{\infty}(u_1, v_2) = \nu_{B_i}(u_1,w) \wedge \nu_{B_i}(w,v_2)$

i.e., either (u_1,w) or (w, v_2) or both are the weakest B_i-edges of the above B_i-cycle because P_i is as strong as u_1,w, v_2.

This is not possible because u_1,w, v_2 is a strongest B_i-path.

Therefore, w is a B_i -cut vertex.

Now we prove that the internal vertices of a B_i -tree of an IF B_i-tree are the B_i-cut vertices.

Theorem (3.14): Let $\tilde{G}$ be an intuitionistic fuzzy B_i -tree for which $\tilde{F}_i$ = (A,C_1,C_2,...,C_k) is a partial IF spanning subgraph structure which is a C_i-tree and $\mu_{B_i}(x,y) < \mu_{C_i}{}^{\infty}(x,y)$ and $\nu_{B_i}(u,v) > \nu_{C_i}{}^{\infty}(x,y)$ ∀(x,y) not in $\tilde{F}_i$. Then the internal vertices of $\tilde{F}_i$ are precisely the B_i –cut vertices of $\tilde{G}$.

Proof: Consider a vertex w of $\tilde{F}_i$.

Case I: w is not an end vertex of $\tilde{F}_I$,

Therefore, w is common to two C_i-edges of $\tilde{F}_i$ at least and by Theorem (3.7), they are B_i-bridges of $\tilde{G}$. Then by Theorem (3.13), w is a B_i-cut vertex.

Case II: w is an end vertex of $\tilde{F}_i$

If w is a B_i -cut vertex, it lies on every strongest B_i-path and hence C_i-path joining u and v for some u and v in V. One of such C_i-paths lies in $\tilde{F}_i$.

But w is an end vertex of $\tilde{F}_i$. So this is not possible.

So w is not a B_i-cut vertex i.e., the internal vertices of $\tilde{F}_i$ are precisely the B_i-cut vertices of $\tilde{G}$.

The above theorem leads us to the following corollary.

Corollary (3.15): A B_i-cut vertex of an intuitionistic fuzzy graph structure $\tilde{G}$ which is an intuitionistic fuzzy B_i-tree, is common to at least two B_i -bridges.

Acknowledgement: The second author would like to thank IKG PT University, Jalandhar for providing an opportunity to do research work under her supervisors.

International Journal of Engineering, Science and Mathematics (UGC Approved)
Vol. 6 Issue 8, December 2017, ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

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